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Wavelet transform: a future of rock fabric analysis?

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Abstract

Although twenty years ago fabric was defined as "the complete spatial and geometrical configuration of all those components that make up a deformed rock", fabric was mainly synonymous with lattice preferred orientation. Very little attention has been paid to the multi-scale geometrical and spatial relationships of the rock components. Fabric quantification, in terms of size, shape, orientation and location, at all scales, is now performed in two dimensions using anisotropic wavelets. As a first example, the wavelets are applied to the famous colour plate of Sander, and the results compared to his Axial Distribution Analysis. Applied to the K-feldspars of a rock section from the Sidobre granite pluton (Montagne Noire, France), the wavelet analysis shows: (i) alignment of grains, resulting from mechanical interactions between grains, and shows that shearing occurred within the crystalline frame; (ii) preferred grain orientation, the classical grain-shape fabric, parallel to the overall mineral lineation; and (iii) small tensional domains, elongate perpendicular to the lineation, infilled by the residual melt just before total crystallisation, attesting to the stretching nature of the mineral lineation. The future of rock fabric analysis will come from new steps in understanding the processes acting during fabric development along with a further development of wavelet analysis using high resolution three-dimensional fabric data.

1. Background of fabric analysis

Pioneering fabric studies (Gefüge) of Sander (1911, 1970), were primarily devoted to quantifying the orientations of crystal lattices (lattice preferred orientation, LPO) in metals and rocks, and aimed at understanding the micro-mechanisms of working conditions or plastic deformation experienced by these materials under artificial or natural conditions. In conjunction with experiments (Tullis, 1977) and numerical simulations (Lister and Paterson, 1979; Lister and Hobbs, 1980), LPO studies have mainly been used to infer the sense of shearing undergone by strongly deformed peridotites (Nicolas et al., 1973), quartzites (Bouchez, 1977) and ice (Hudleston, 1977; Bouchez and Duval, 1982). Increasingly sophisticated techniques have been developed for LPO studies, on grounds of both data acquisition by U-stage, X-ray and neutron goniometry (Mainprice et al., 1993), and numerical techniques for fabric representation such as the Orientation Distribution Function (ODF; Bunge, 1981; Schmid et al., 1981), giving the complete description of the lattice orientation of a crystalline material.

During the period of these LPO studies, shape fabric or SPO studies remained in relative infancy. SPO studies were mainly confined to the determination of the average grain elongation (lineation) and flattening (foliation) in order to set the structural framework with respect to which the LPO was reported (Benn and Allard, 1989). In other words there was no obvious need for detailed SPO studies in the structural geology of rocks deformed in the solid state. The increasing interest of structural geologists in magmatic rocks and their mechanisms of flow forced them, however, to further develop fabric studies.

A little more than twenty years ago, Hobbs et al. (1976) generalised the concept of fabric by integrating the previous works in a more general definition where fabric is defined as the "complete spatial and geometrical configuration of all those components that make up a deformed rock" (Bates and Jackson, 1980,

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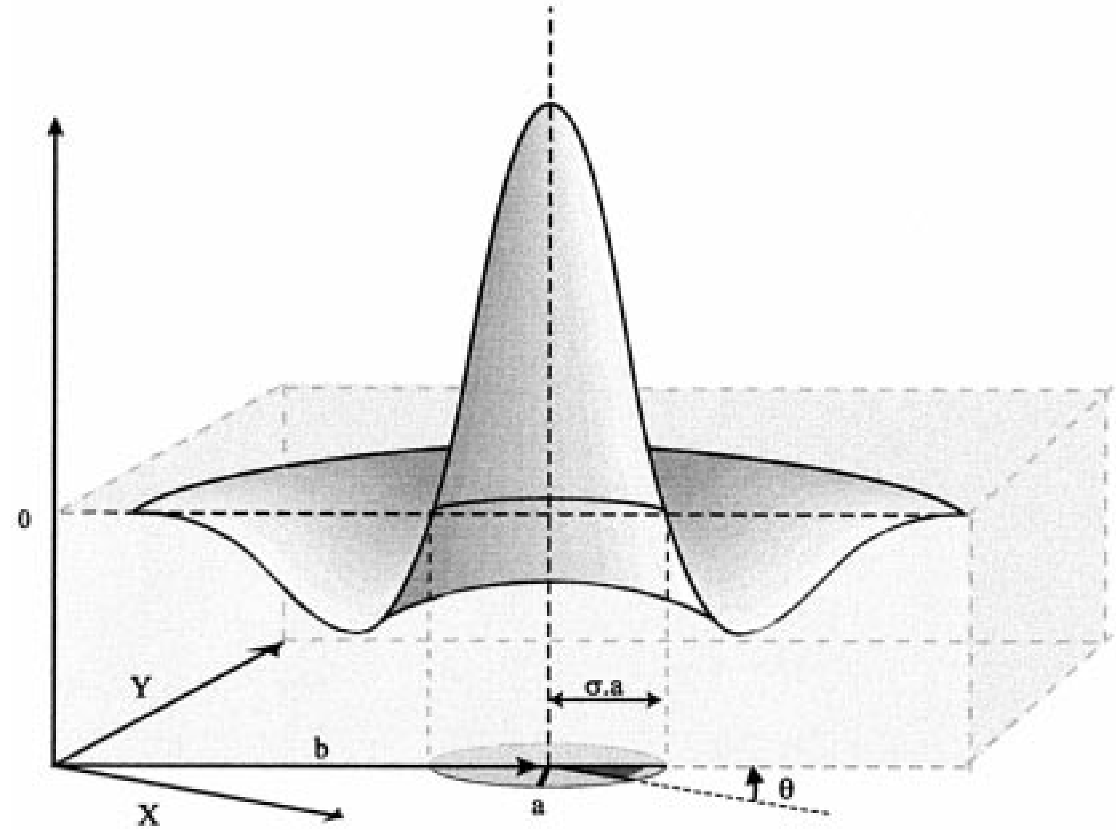

Fig. 1. Schematic cross-section of an anisotropic wavelet filter $\psi(a,\mathbf{b},\sigma,\theta)$. Parameters are: resolution (a), location (**b**), shape ratio (σ) and orientation (θ) in the cartesian coordinate system (X, Y). The detected entity is characterised by the following parameters: location (b), half short axis (a), half long axis ($\sigma.a$), and azimuth of the long axis (θ).

p. 220). This definition stresses the importance of the size of the field of investigation, or scale, which in turn depends on the particular domain of interest to which the fabric study is applied. In order to discriminate the physical processes at the origin of the fabric of an observed rock, a complete description of its fabric is necessary. This includes quantification of the individual components of the rock mass and recognition of the particular organisations resulting from a non uniform spatial distribution of its components. For example, clusters or alignments of interacting grains may reveal the role of mechanical interactions between grains in a ductile matrix (Blumenfeld and Bouchez, 1988; Tikoff and Teyssier, 1994). Patterns of clustered grains may provide information about depositional processes in sedimentary rocks, or about the control of nucleation in igneous and metamorphic rocks (Morishita and Obata, 1995; Jerram et al., 1996).

Up to now, only the orientation distribution of grain elongations in two-dimensional sections (i.e. SPO) has been concretely studied, benefiting from the conceptual and technical advances due to computers and development of image processing techniques (e.g. Panozzo, 1987; Launeau and Robin, 1996, among others). Spatial distribution analyses remained restricted to specific problems such as intergranular diffusion and mechanisms of porphyroblast crystallisation using pattern analysis (Shehata, 1989; Carlson et al., 1995), analysis of small scale structure and porosity of sandstone using Fourier power spectra (Prince et al., 1995) and rock deformation using autocorrelation function (Panozzo Heilbronner, 1992; Pfleider and Halls, 1993). However, geometrical (including size, shape and orientation) *and* spatial distributions, at all scales, have never been examined at the same time. This is now realised using the wavelet transform formalism.

2. Wavelet analysis and NOAWC method

Wavelet analysis has emerged as a powerful tool for the processing of signals in which different scales are combined (Antoine et al., 1993). Mathematical developments of wavelets and some applications are given in Meyer and Roques (1993). Based on the algorithm of Ouillon et al. (1996), Darrozes et al. (1997) have developed the Normalised Optimised Anisotropic Wavelet Coefficient method (NOAWC) which permits detection and quantification of any organised feature

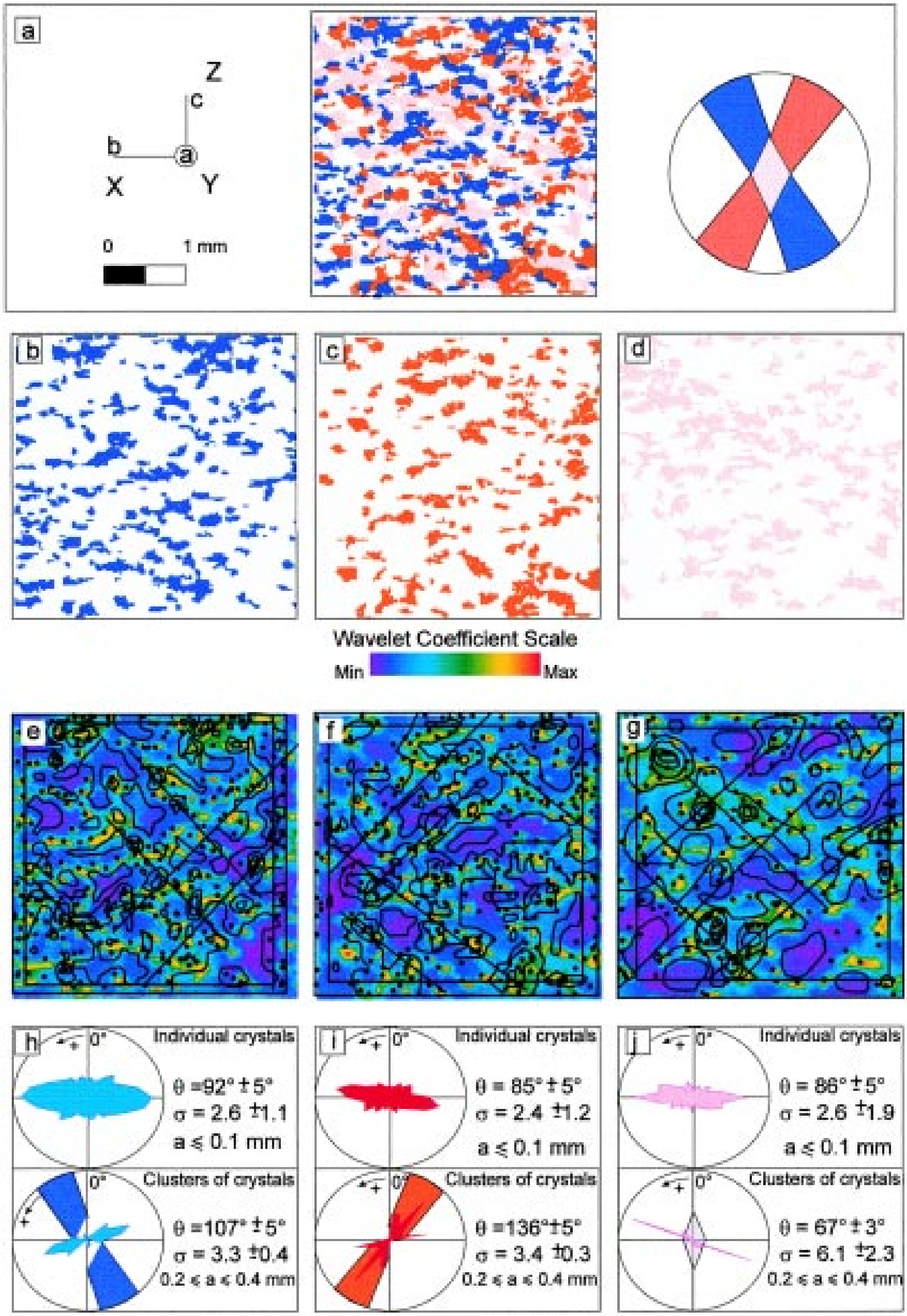

Fig. 2. Axial Distribution Analysis. (a) Gries am Brenner—quartzite in XZ-section (section perpendicular to a). Reproduction and classification of the colour plate of Sander (1950, p. 409) and crossed-girdle type fabric diagram of c-axis. Colours correspond to the orientations of the grain c-axes: blue for NW–SE, red for NE–SW, pink for normal to plane c-axes (Y-maximum), and white for the remaining orientations [this colour is not studied in this work because it is associated with unclassified pixels ($<5\%$)]. (b, c and d): respectively, isolated blue, red and pink sub-populations; (e, f and g) comparison of the Axial Distribution Analysis (AVA) results of Sander (1950, p. 409), represented by the black dots and contours, and the NOAWC results for, respectively, the blue (e), red (f) and pink (g) sub-populations. Notice that AVA domains contoured by Sander correspond to the high wavelet coefficients (in red), underlining grain alignments or clusters. (h, i and j) quantitative description of the crystals and clusters of crystals for, respectively the blue (h), red (i) and pink (j) sub-populations. Notice that individual grains are mainly elongate parallel to X with a mean shape anisotropy of about 2.5, and that clusters are oblique to the XY baseline and are more anisotropic ($\sigma > 3.0$). Published with kind permission from Springer-Verlag, Berlin.

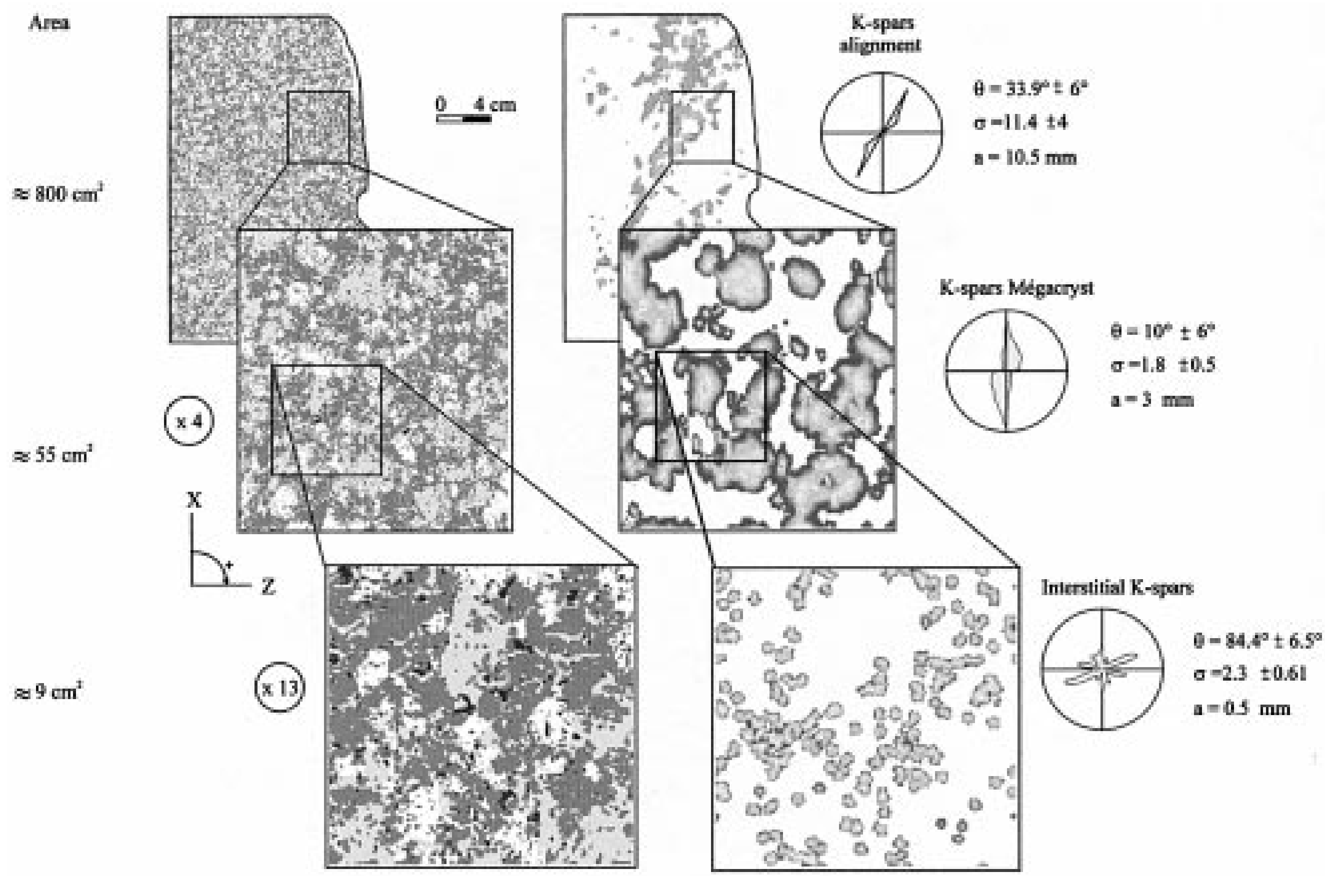

Fig. 3. Fabric analysis of K-feldspars (light grey) in a granite. *XZ* coloured rock section (left) of the Sidobre granite (Montagne Noire, France) with quartz (white), plagioclase (dark grey), K-feldspars (light grey) and biotite (black). The results of the multi-scale analysis using the NOAWC are shown on the right. At large scale ($a = 10.5$ mm), alignments of grains around 34°, resulting from mechanical interactions between grains, and slip at the matrix/grain interfaces attest to shear movement in the crystal frame at a relatively low liquid fraction. At the grain scale ($a = 3$ mm), the well-defined grain shape preferred orientation around 10° underlines the stretching direction. At small scale ($a = 0.5$ mm), interstitial veinlets or worm-like spaces perpendicular to *X* are attributed to late crystallisation processes, with infilling of microfractures in the presence of melt.

of a binary image (objects, clusters or alignments of objects) regardless of its scale, shape, orientation and location.

With NOAWC the whole image is analysed by a range of anisotropic wavelet filters $\psi(a,\mathbf{b},\sigma,\theta)$, all derived from a mathematical mother function called the 'Anisotropic Mexican Hat' (Ouillon et al., 1995, among others). These filters depend on four variables corresponding to the four parameters of any object (or group of objects) (Fig. 1): (i) the scale parameter, or resolution (a), allows multi-scale analysis by dilatation/contraction of the filter; (ii) the translation vector (**b**) defines the location of the filter on the image and therefore permits exploration of the whole image; (iii) the shape ratio (σ) and (iv) the azimuth (θ) of the long axis of the wavelet allows detection of any anisotropic feature in any direction (Darrozes et al., 1997). So, the complete quantification of all the organisations of a binary image by NOAWC allows examination of a complex image, by simultaneously exploring sizes, shapes, orientations and spatial relationships of constituents at all scales. Gaillot et al. (1997) recently gave an example of the efficiency of this method using images of both a synthetic and a natural rock fabric.

3. Axes Distribution Analysis

This example is taken from the famous plate of Sander (1950, p. 409) reproduced in Fig. 2(a). It represents the photomicrograph of a quartzite in *XZ* section (parallel to grain elongation, *b* of Sander, and perpendicular to grain flattening, *c* of Sander) with an elongate mosaic microstructure and a strong lattice fabric of the quartz grains. The fabric diagram of *c*-axes is of the classical crossed-girdle type (Lister, 1977), and the plate is coloured according to the orientation of the grain *c*-axes (Fig. 2a).

Analysis of the spatial domains of similar orientations, or Axes Distribution Analysis of Sander (1970,

p. 256), may be achieved by analysing separately the domains of similar colours. This has been performed by Ramsauer in Sander, for the blue, red and pink orientations, by pointing the centres of gravity of grains having the same colour and by contouring into domains according to the density of dots (black dots and contours of Fig. 2e–g).

The wavelet analysis of the same plate is also represented in Fig. 2. In Fig. 2(e–g), the very low NOAWC coefficient intensities are in blue, very high intensities are in red. In Fig. 2(h–j), the fabric quantification is presented as rose diagrams and geometrical parameters, for both individual grains (small resolutions) and clusters or alignments of grains (large resolutions). Fig. 2(e–g) show that the AVA domains contoured by Sander correspond to high intensity domains of the wavelet coefficients, underlining grain alignments or clusters. Because every resolution (a) and every anisotropy (s) is explored by NOAWC, the preferred shape orientations of individual crystals and clusters of crystals (Fig. 2h–j) can be extracted from the coefficient map. In short, this map indicates that the individual grains (for which the average shape ratios are given) are precisely elongate parallel to X of the frame ($\theta = 85–92° \pm 5°$), and that the clusters are oblique to the XY baseline ($\theta = 67–136°$). These obliquities represent precisely what Sander examined with the AVA, and interpreted in a quite complex fashion. Although it is out of the scope of this paper to discuss the interpretation, the oblique trends of the blue and red grains could be due to the dismembering of ancient large porphyroclasts of quartz grains parallel to subgrains that were either basal (Fig. 2i) or prismatic (Fig. 2j) in orientations, and that evolved later into grain boundaries.

4. Fabric analysis of K-feldspars in a granite

For a better understanding of granite fabric evolution from the magmatic state to the solid state, the following example illustrates the case of an XZ section (parallel to magmatic lineation and perpendicular to magmatic foliation) of a porphyritic granite from the Sidobre pluton (Darrozes et al., 1994; Gaillot et al., 1997). The section has been etched and coloured by chemical means based on the technique of Bailey and Stevens (1960) in order to separate the main phases: Na-rich plagioclase in red (due to amaranth), K-feldspars in yellow (due to cobaltnitrite), quartz remaining in white and biotite in brown. These colours helped in classifying and segmenting the image in four grey levels (Fig. 3, left).

Textural evolution of the granite just before its complete crystallisation is studied through the fabric of the K-feldspars which present two different habits: the (mega)crysts that crystallise at a relatively early stage, and the interstitial K-feldspars which represent one of the very late phases to crystallise. (i) For an observation field of $800\ cm^2$ (Fig. 3: upper left), NOAWC detects, with a resolution $a = 10.5$ mm, alignments of crysts which are 11.5 cm in average length ($\sigma = 11$) and disposed obliquely ($\theta = 34°$) with respect to the foliation (Fig. 3: upper right). These alignments are interpreted as resulting from mechanical interactions between groups of K-feldspar crystals, among other phases, and slip at the grain–matrix interfaces (Ildefonse et al., 1992; Ildefonse and Mancktelow, 1993). They attest to shear movement in the crystal frame at relatively low liquid fractions. (ii) A closer view is given by studying a $55\ cm^2$ subarea taken randomly from the former field: a resolution $a = 3$ mm, given by the maximum intensities of the NOAWC coefficients, yields detection of the individual crystals whose average length is 5.4 mm ($\sigma = 1.8$) and orientation at $10° \pm 6°$ from the X-axis. (iii) Finally, a still closer inspection of (almost) any subarea of $9\ cm^2$ extracted from the latter image yields, at a critical resolution $a = 0.5$ mm, of small elongate ($\sigma = 2.3$) interstitial veinlets or worm-like spaces that are strongly oblique to X ($\theta = 84.4° \pm 6.5°$). These features represent the infilling by the melt remaining in the rock just before total crystallisation of intergranular voids, or intragranular microfractures (Bouchez et al., 1992).

The observed perpendicular-to-X nature of these small tensional features strongly supports the 'macroscopic' mineral lineation X pointing to the stretching direction to which the rock was subjected during, at least, the late stages of magma straining. This also agrees with field studies showing that, in the Sidobre pluton (Darrozes et al., 1994) or Elba Island pluton (Bouillin et al., 1993), the orientations of the late aplite dikes are, on average, normal to the magmatic lineation. Along with the results previously obtained on the Sidobre pluton (Améglio et al., 1994; Darrozes et al., 1994), the spatial and geometrical configurations of the K-feldspar crystals help to propose a continuum of extensional deformation from the magmatic state to the solid state.

5. Conclusion and perspective

Being a complex record of the processes which have acted in the rock, a fabric corresponds to the stacking in three dimensions of mineral organisations at various scales. A reliable depiction of this record requires fabric quantification of object sizes, shapes, orientations and locations at all scales, in addition to comparisons with experiments and models of rock forming processes. We have shown that image exploration with anisotropic wavelets, and extraction of parameters linked with maximum wavelet coefficients (NOAWC), yield the spatial and geometrical relationships of objects of a given mineral phase at all scales. However, our approach has been restricted to two-dimensional binary images of rock sections. Until now, extension to three dimensions has been mainly constrained by requirement of: (i) high resolution input data for several mineral phases and large volumes of rock; and (ii) expensive computer time of such data analysis.

On the one hand, recent acquisition of three-dimensional complete fabric data by means of high-resolution computed X-ray tomography (Denison and Carlson, 1997) offers new potentials for fabric analysis. From the sizes and locations of thousands of crystals, the degree of spatial order, clustering, intergrowth and relative isolation of porphyroblasts have been used to identify the mechanisms governing nucleation and growth of porphyroblasts (Denison et al., 1997). On the other hand, due both to the flexibility of the Wavelet Transform formalism (Antoine and Murenzi, 1994; Hagelberg and Helland, 1995), and the increasing power of computers, extension of NOAWC to three-dimensional colour images will soon be available for three-dimensional quantification of polyphased rock fabrics. Particularly with three-dimensional data, wavelet transform opens new directions for rock fabric analysis and for the knowledge of rock deformation processes.

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